**Alternating Series, Absolute and Conditional Convergence**

**Theorem:** The alternating series test (Leibniz’s test)

The series



converges if all three of the following conditions are satisfied

1. The ’s are all positive.
2. The positive ’s are decreasing:

 for all .

1. .

**Example:**

Which of the following series converge, and which diverge

(1)  (2) 

**Solution:**

(1) 



(i) ’s are all positive

(ii) if 

then is decreasing and so is decreasing.

(iii) 

then all three conditions are satisfied and so  converges.

(2) 



(i) ’s are all positive

(ii) if .

then is decreasing and so is decreasing.

(iii)

then all three conditions are satisfied and so  converges.

**Definition:**

A seriesconverges absolutely (is absolutely convergent) is if the series converges .

**Theorem:** (The absolute convergence test)

If converges, then converges.

**Remark:**

The converse statement of the above theorem is false. For example;

in above example we show that  is converges, but the series  is divergent (p-series and).

**Example:**

Prove that the series is absolutely convergent.

**Solution:**



 is convergent because (*p*-series), then the series  is convergent and so is absolutely convergent.

**Definition:**

The series  is conditional convergence if the series converges but the series  diverges.

**Example:**

Prove that the series  is conditional convergence.

**Solution:**

 is divergent because  (*p*-series).

Then the series is not absolutely convergent. Now we discuss the convergence of 



(i) ’s are all positive

(ii) if 

then is decreasing and so is decreasing.

(iii) 

then all three conditions are satisfied and so  converges. Then the series  is conditional convergence.

**Exercises**

State whether the following series absolutely convergence, conditional convergence or divergent ?

         